

Simulations of control for Navier-Stokes equations

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In many areas of science, we aim to drive a system (biological, chemical, physical, ...) to a desired state or behavior. The ability to do this in a manner is called controllability. In this communication, we address the problem of controlling the wake of an incompressible fluid past an obstacle (in 2D), by blowing-suction localised on the boundary of the obstacle.

More precisely, we consider the incompressible Navier-Stokes equation on $\Omega \subset \mathbb{R}^2$, an open bounded set, with control localised on a part of the boundary Γ , to stabilize the fluid on an unstable stationary state.

The velocity z and the pressure q of a (newtonian) incompressible viscous fluid inside the domain Ω satisfy

$$\begin{cases} \dot{z} + (z \cdot \nabla)z - \nu \Delta z + \nabla q = 0, \\ \text{Div}(z) = 0, \end{cases}$$

together with suitable initial and boundary conditions.

We consider the following configuration, from the CARPE Project¹.

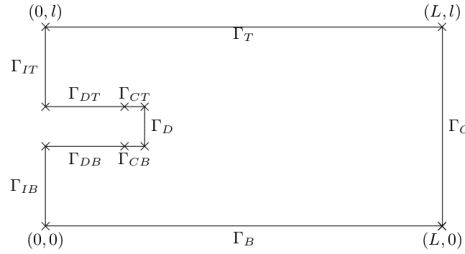


Figure 1: The domain Ω .

The boundary conditions are the following. On the inlet : $z_1 = g$ and $z_2 = 0$ on $\Gamma_{IT} \cup \Gamma_{IB}$, with a suitable profile g , chosen to be an approximation of the Blasius boundary layer profile. On the outlet: $\sigma(z, q) \cdot n = 0$ on $\Gamma_T \cup \Gamma_O \cup \Gamma_B$, where σ is the stress tensor. We complete boundary conditions by the Dirichlet one. In the uncontrolled case, this will only be homogeneous Dirichlet boundary condition. In the controlled case, we will apply a control on Γ_{CT} and Γ_{CB} , that is non-homogeneous Dirichlet condition.

In this work, we will briefly present the theoretical result of Raymond [1], which says that our problem is well-posed and gives a way to compute the (optimal) control (which will be of feedback form) and then focus on the discretization of the problem.

Références

- [1] RAYMOND, JP, *Feedback boundary stabilization of the two dimensional navier-stokes equations*, (2006), SIAM Journal on Control and Optimization, 45(3), 790828.

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¹The CARPE Project